Time and Frequency Domain Two-tone Simulation based on Periodic Steady State Linearization

Maysa A. C. Araújo [1], Dhessica M. S. de Moura[2], Eduardo G. de Lima [3]
Department of Electrical Engineering, UFPR
Group of Integrated Circuits and Systems
Curitiba, Brazil
maysaalves@ufpr.br [1], dhessica.moura@ufpr.br [2], elima@eletrica.ufpr.br [3]

***Abstract*—In a nonlinear electronic circuits analysis, voltages and currents are related by a network of nonlinear equations. One of the most used methods to process the solution is to find a linearized model around a DC operation point. However, the harmonic distortion contribution is despised, and when the variations of the amplitude are bigger, the harmonics cause distortions in voltages and currents wave-forms. Therefore, using as a case of study a test circuit of a power amplifier, excited by a two-tone sinusoidal voltage source, which the nonlinear element is a voltage controlled current source, this paper study two approaches of linearization and simulation: Harmonic Balance (HB), in the frequency domain, and Shooting Method (SM), in the time domain and the linearity valid limits are analyzed.**

***Index Terms*—nonlinearities, time domain, frequency domain, shooting method, harmonic balance**

I. Introduction

Modeling and simulation of electronics circuits play an important role in designing and fabricating electronics equipment, providing the analysis and solving real-world problems in a safe and efficient way, as prevent circuit designers from spending time and resources to accomplish their design specifications. Usually, in circuits containing independent sinusoidal sources, just the steady state is obtained from simulations, due to the absence of knowledge about the initial conditions and to avoid the additional complexity demanded by the transient response.

The importance of an accurate and efficient steady state analysis can be exemplified by the verification of some performance parameters of radiofrequency (RF) circuits that need to be computed in steady state, such as distortion, power consumption, frequency, noise and transfer characteristics [1]. Steady state analysis methods can be divided as frequency and time domain methods. Both steady state methods are supported by many commercial simulators with compact device models. The time domain Shooting Method (SM) and the frequency domain Harmonic Balance (HB) have been developed for coupled device and circuit simulation. So, a comparison of the performance of the two methods in the context of coupled device and circuit simulation is important, for example [1].

Large and small-signal analyses are commonly applied in design of mixers and power amplifiers and in nonlinear noise analysis. The technique is based by seeking the linear response to a much smaller signal applied in circuits with a nonlinear element driven by a single large sinusoidal signal. At first, the nonlinear device is analyzed under large signal excitation only, generally by the HB method. Then, the device is linearized in order to create time-varying, linear, small-signal elements and at last, a small-signal analysis is made [2].

In the small-signal analysis, the linearization around the value obtained by direct current (DC) analysis does not suffer an impact on the response accuracy. However, for large signals, when linearization is performed around the DC response, it is not possible to guarantee result coherence, because in this case the harmonics generated by the system influence the voltage amplitudes and circuit current [3].

This paper presents a comparison between the methods in frequency domain, using HB, and time domain, using SM. The linearity valid limits are also discussed.

II. Periodic Steady State Analysis

This nonlinear analysis computes the response of the circuit with a simulation time independent of the initial conditions. The steady state response is obtained directly, not requiring to simulate the transient behavior of the circuit hence avoiding the many cycle time integration [4].

Some techniques used to solve in this approach will be described in the next subsections, such as HB and SM.

A. Harmonic Balance

The HB method is a frequency domain approach used to analyze the distortion of nonlinear devices. This method uses the Kirchhoff's Current Law at each node and for each independent frequency. One unknown is defined as the DC component while the others are related to the fundamental and the harmonic amplitudes, and a certain number of harmonics is considered. The circuit differential equations are expressed in terms of the Fourier coefficients and the differentiation in time domain is replaced by algebraic multiplication in frequency domain [5].

The algebraic system of nonlinear equations is solved once. However, it is recommended to not use it in circuits with a great amount of nonlinearities, due to the increase of harmonics and hence the increase of the system order.

B. Shooting Method

The SM is a numerical procedure based in a time domain analysis, by reducing the system to an initial value problem for solving a boundary value problem [4].

Given a circuit with dynamic elements, the SM finds the solution of an algebraic nonlinear system from initial values that provide a steady state condition. When simulating one period of the fundamental frequency, the last value must be the same as the guessed initial state. This approach uses the transient analysis and the differential equations are discretized for being integrated one timestep at time. Furthermore, the next step of the iteration depends on the previous values.

III. Linearization around a Periodic Steady State

The linearization technique can be applied for different electronic circuit analysis method. As general rule, it is necessary to differentiate the element equation in relation to its initial state for linearizing it. The following equation describes the linearization of an unknown [2]:

$$f_{NL}(X) = \frac{df_{NL}(X)}{dX}|_{X=X_0} \cdot (X - X_0), \tag{1}$$

where $X_0(t)$ is the initial state and $f_{NL}(X)$ is the nonlinear function of the unknown $X(t)$ to be linearized. In this work, $X_0(t)$ is the periodic steady state response to a large-signal one tone excitation. The linearized circuit is then also stimulated by a small-signal second tone.

A. Frequency Domain

The frequency domain is known in literature as Periodic Alternating Current (PAC). The application of the PAC method, which uses HB method, linearizes the analyzed circuit around the steady state response to large-signal one tone excitation.

When applying the HB method, the initial trajectory function to be used on linearization is composed of one constant corresponding to the DC component, plus the harmonic contributions that alternate depending on the numbers of configured harmonics (H), as exemplified in:

$$x_{hb}(t) = x_0 + \sum_{h=1}^{H} x_{hs} \sin(h\omega_0 t) + x_{hc} \cos(h\omega_0 t), \tag{2}$$

where x_0, x_{hs}, and x_{hc} are constants. The Equation (2) can be reorganized in order to become a multiplication of two vectors, one vector consisting of periodic time functions and one amplitude vector composed of the DC component, plus the configured harmonic contributions, as exemplified in:

$$x_{hb}(t) = \overrightarrow{X_{HB}} \bullet \overrightarrow{Cont_{HB}}(t), \tag{3}$$

$$\overrightarrow{X_{HB}} = [x_0 \quad x_1 \quad \cdots \quad x_{2H+1}], \tag{4}$$

$$\begin{aligned}\overrightarrow{Cont_{HB}(t)} = [1 \sin(\omega_1 t) \quad \cos(\omega_1 t) \quad \sin(2\omega_1 t) \\ \cos(2\omega_1 t) \quad \cdots \quad \sin(H\omega_1 t) \quad \cos(H\omega_1 t)]^T.\end{aligned} \tag{5}$$

The voltage or current values to be obtained at the end of the analysis are calculated by applying the superposition method. This method consists of adding the initial trajectory $x_{hb}(t)$ with a linearized component $x_{lin}(t)$. To get $x_{lin}(t)$ it is necessary to differentiate the nonlinear component function around $x_{hb}(t)$. The obtained response can be reorganized in a multiplication of an amplitude matrix $\overrightarrow{(G_{HB})}$ and a column vector $\overrightarrow{(X_{1tone})}$ composed of the positive and negative fundamental component of the second tone and the harmonic contributions of the first tone, as exemplified in:

$$x_{lin}(t) = \overrightarrow{G_{HB}} \bullet \overrightarrow{X_{1tone}}(t), \tag{6}$$

where $\overrightarrow{G_{HB}}$ truncated to a 10x10 matrix is equal to:

$$\begin{vmatrix}
g_0 & 0 & \frac{g_2}{2} & \frac{-g_1}{2} & \frac{g_4}{2} & \frac{-g_3}{2} & \frac{g_6}{2} & \frac{-g_5}{2} & \frac{g_8}{2} & \frac{-g_7}{2} \\
0 & g_0 & \frac{g_1}{2} & \frac{g_2}{2} & \frac{g_3}{2} & \frac{g_4}{2} & \frac{g_5}{2} & \frac{g_6}{2} & \frac{g_7}{2} & \frac{g_8}{2} \\
\frac{g_2}{2} & \frac{g_1}{2} & g_0 & 0 & \frac{g_2}{2} & \frac{-g_1}{2} & \frac{g_4}{2} & \frac{-g_3}{2} & \frac{g_6}{2} & \frac{-g_5}{2} \\
\frac{-g_1}{2} & \frac{g_2}{2} & 0 & g_0 & \frac{g_1}{2} & \frac{g_2}{2} & \frac{g_3}{2} & \frac{g_4}{2} & \frac{g_5}{2} & \frac{g_6}{2} \\
\frac{g_4}{2} & \frac{g_3}{2} & \frac{g_2}{2} & \frac{g_1}{2} & g_0 & 0 & \frac{g_2}{2} & \frac{-g_1}{2} & \frac{g_4}{2} & \frac{-g_3}{2} \\
\frac{-g_3}{2} & \frac{g_4}{2} & \frac{-g_1}{2} & \frac{g_2}{2} & 0 & g_0 & \frac{g_1}{2} & \frac{g_2}{2} & \frac{g_3}{2} & \frac{g_4}{2} \\
\frac{g_6}{2} & \frac{g_5}{2} & \frac{g_4}{2} & \frac{g_3}{2} & \frac{g_2}{2} & \frac{g_1}{2} & g_0 & 0 & \frac{g_2}{2} & \frac{-g_1}{2} \\
\frac{-g_5}{2} & \frac{g_6}{2} & \frac{-g_3}{2} & \frac{g_4}{2} & \frac{-g_1}{2} & \frac{g_2}{2} & 0 & g_0 & \frac{g_1}{2} & \frac{g_2}{2} \\
\frac{g_8}{2} & \frac{g_7}{2} & \frac{g_6}{2} & \frac{g_5}{2} & \frac{g_4}{2} & \frac{g_3}{2} & \frac{g_2}{2} & \frac{g_1}{2} & g_0 & 0 \\
\frac{-g_7}{2} & \frac{g_8}{2} & \frac{-g_5}{2} & \frac{g_6}{2} & \frac{-g_3}{2} & \frac{g_4}{2} & \frac{-g_1}{2} & \frac{g_2}{2} & 0 & g_0
\end{vmatrix}, \tag{7}$$

and:

$$\overrightarrow{X_{1tone}}(t) = \begin{vmatrix}
x_8 \sin[(\omega_2 - 2\omega_1)t] \\
x_9 \cos[(\omega_2 - 2\omega_1)t] \\
x_4 \sin[(\omega_2 - 2\omega_1)t] \\
x_5 \cos[(\omega_2 - \omega_1)t] \\
x_0 \sin(\omega_2)t \\
x_1 \cos(\omega_2 t) \\
x_2 \sin[(\omega_2 + \omega_1)t] \\
x_3 \cos[(\omega_2 + \omega_1)t] \\
x_6 \sin[(\omega_2 + 2\omega_1)t] \\
x_7 \cos[(\omega_2 + 2\omega_1)t]
\end{vmatrix}, \tag{8}$$

where $g_0, \cdots, g_8$ are constants that indicate the HB amplitude components of the unknown manipulated by the nonlinear element function.

B. Time Domain

To make the nonlinear shooting, at first it is necessary to obtain the response of the fundamental frequency f_1 voltage source acting alone, annulling the small-signal source. The voltages and currents obtained are periodic signals for finite timesteps and the function period is $\frac{1}{f1}$. Furthermore, the nonlinear periodic function, which in this paper is a voltage controlled current source, is calculated for this voltage source. The values found for the voltages are used for the next step, for calculating the circuit currents, until it reaches the final step. The final value of each state variable (capacitor voltage or inductor current) is subtracted by the initially assigned value:

$$V_{Bprev} - V_B(t_0) = 0, \tag{9}$$

$$V_{Cprev} - V_C(t_0) = 0, \tag{10}$$

where the unknowns with the subscription "prev" are referred to the values found at the last iteration, and V_B and V_C indicate voltages over capacitors.

After the end of the simulation, the nonlinear function is linearized around the first SM result, thus it is now time-varying with frequency $f1$ whereas the voltage injected by the small-signal source has frequency $f2$. The single-event transient of the equivalent circuit is linear as well and a new SM is processed. As the linearized parameter requires a trigonometric multiplication, the fundamental frequency of the linear system is $\frac{1}{f1-f2}$:

$$I_{lin} = gm(t)[V_{Alin} \sin(2\pi f_2 t)], \tag{11}$$

where V_{Alin} is the linearized voltage amplitude and gm is a time-varying variable with frequency f_1, result of derivative of the nonlinear trigonometric function.

IV. Case Study

The circuit schematic is shown in Figure 1.

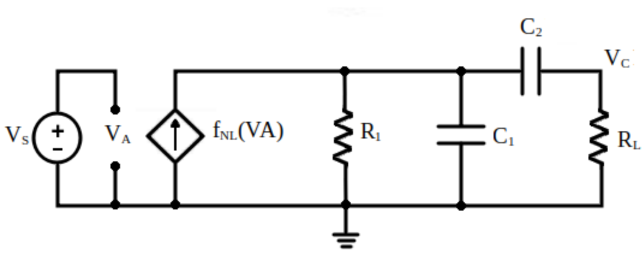

Fig. 1. Circuit Schematic.

In this circuit, V_s is a two-tone sine-wave voltage source, with different amplitudes and frequencies. The circuit parameters are C_1 = 10 pF, C_2 = 1 μF, R_1 = 1 kΩ, R_L = 50Ω, f_1=1 GHz and f_2=1.1 GHz. The nonlinear element is f_{NL}, the equation that describes the nonlinear device behavior is:

$$f_{NL} = \frac{I_{sat} sign(VA)}{(1 + (\frac{V_{sat}}{|VA|})^s)^{\frac{1}{s}}}, \tag{12}$$

where I_{sat} and V_{sat} are the maximum source saturation, VA is a nodal voltage equals to the voltage injected by the independent source, $sign(VA)$ is a sign function and s indicates the wave damping factor. Therefore, the smallest the value of s, the smoother the signal response. The parameters of this voltage controlled current source are: I_{SAT} = 0.1 A, V_{SAT} = 1.8 V and s = 5 for the Envelope Tracking (ET) architecture of a power amplifier. Every parameter is fixed, except the second tone amplitude. The first tone large-signal amplitude is fixed in 1.8 V. The second tone amplitude is either a small-signal of 0.2 V or a large-signal of 1.2 V.

As this study is based to analyze within the limits of the applied methods, and knowing that the critical point is the analysis of small-signals, i.e., the different outputs for variations in the amplitude of the excitation source V_s were observed, using MATLAB® Software for simulating and plotting the wave-forms. In order to make a comparison between the frequency and time domain methods, the Figures 2 and 3 show a SPICE-like simulation, a SM response and a PAC simulation response.

In PAC, two algebraic systems are needed to be solved. The first one is a nonlinear system that originated from the HB analysis. Knowing that H refers to the quantities of harmonics considered, the number of unknowns and equations, in this case, is equal to $2H+1$. The second one is a linear system with the number of unknowns and equations equals to $2(2H+1)$. In this study, it was used $H = 10$.

In SM, the main goal is to linearize the nonlinear circuit being fed by a voltage source with frequency equal to the second tone, around the response obtained by applying SM in the fundamental frequency circuit. Therefore, the result in this approach is a superposition of two sinusoidal sources with different frequencies and amplitudes. For that reason, it is also necessary to solve two algebraic systems. At first, a nonlinear system in which the number of unknowns and equations is equal to the number of the state variables, which in this study, are two, and then a linear system with two equations as well.

Finally, it was processed a SPICE-like simulation, with the independent source injecting simultaneously both tones, and knowing this method always works, this will be used to assess the linearization validity limit, making a comparison between both PAC and SM wave-forms results.

The Figures 2 and 3 demonstrate that both methods present very similar results. Therefore, it is more useful to analyze which method requires less calculus and computer processing.

As can be seen in Figure 3, with the second tone amplitude equals to 1.2 V, it is notable that bigger the voltage amplitude applied to the second tone, worse the signal response behavior. In that case, instead of SPICE-like simulation, an HB with artificial mapping [3] or a quasi-periodic steady state (QPSS) [6] could also be adopted.

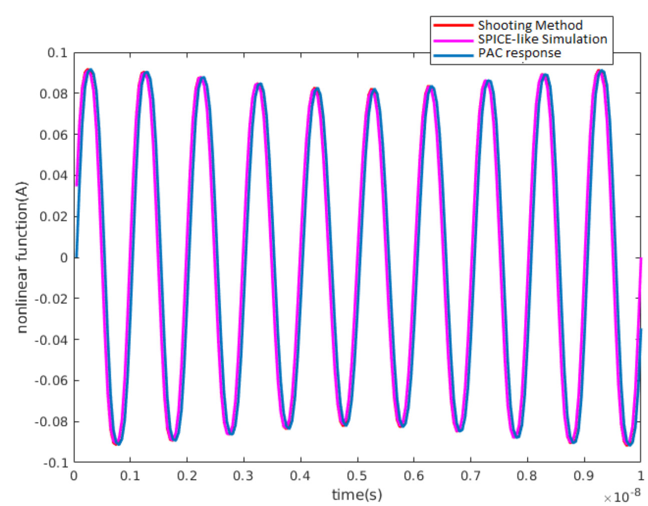

Fig. 2. Simulation response for second tone amplitude equal to 0.2 V.

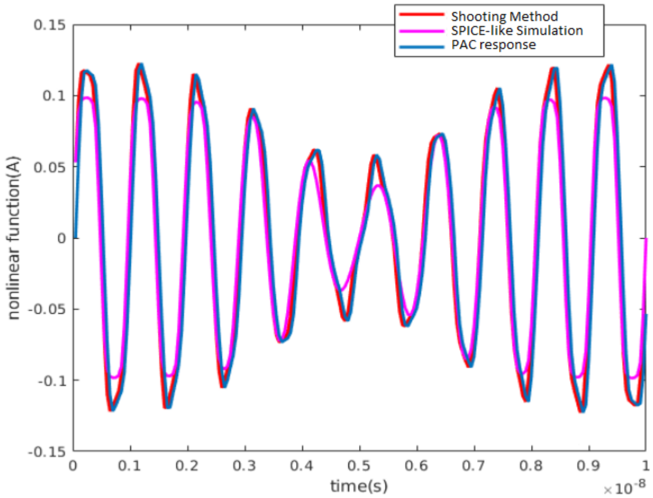

Fig. 3. Simulation response for second tone amplitude equal to 1.2 V.

V. Conclusion

In both time and frequency domain approaches, the circuit is linearized around a one-tone. It is used large-signal in the first tone, but it is only possible to use small-signal in the second tone.

Upon the method calculus and computer effort being compared, the number of unknowns is higher for PAC, however, the complexity of SM is increased because it demands for one fundamental period transient analysis.

As can be seen in the simulation results, when the voltage amplitude in the second tone is increased, it is observed a greater amount of distortions. The nonlinear function presents bigger values than scale current. Furthermore, there is a big difference between the SPICE-like simulation and the SM and PAC response. It follows that linearization is valid in the first case, with smaller values of amplitude, but not for bigger values.

VI. Acknowledgment

This study was financed by National Council for Scientific and Technological Development (CNPq).

References

[1] Y. Hu, X. Duan and K. Mayaram. ”A comparison of time-domain and HB steady state analyses for coupled device and circuit simulation,” The 2nd Annual IEEE Northeast Workshop on Circuits and Systems, 2004. NEWCAS 2004., Montreal, Quebec, Canada, 2004.

[2] D. M.S. Moura, C. G. Natalino and E. G. Lima,”Linearization of eletronic circuits around steady state in zero, one and two tones.” SeMicro-PR Transl. ”Linearização de circuitos eletrônicos em torno de regime permanente a zero, um e dois tons”. Curitiba, Brazil, 2019.

[3] S. MAAS. ”Nonlinear microwave and RF circuits”, Artech House, 2003.

[4] O. Nastov, R. Telichevesky, K. Kundert and J. White, “Fundamentals of fast simulation algorithms for RF circuits,” Proc. IEEE, vol. 95, no. 3, Mar. 2007, pp. 600-621.

[5] K. Mayaram, D. C. Lee, S. Moinian, D. A. Rich and J. Roychowdhury, ”Computer-aided circuit analysis tools for RFIC simulation: algorithms, features, and limitations,” in IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing, vol. 47, no. 4, pp. 274-286, April 2000, doi: 10.1109/82.839663.

[6] K. KUNDERT, J. WHITE, AND A. SANGIOVANNI-VINCENTELLI, A mixed frequency-time approach for distortion analysis of switching filter circuits, IEEE J. Solid State Circuits, 24 (1989), pp. 443-451.